and, finally

$$\begin{cases} \theta_3 = q_1 + (K' + 1)Qq_2 = q_1 + (J_3/I_1)q_2 \\ \psi_3 = q_1 - (K + 1)q_2 = q_1 - (I_3/I_1)q_2 \end{cases}$$

The amplitude of q_2 decreases as $e^{-\lambda\tau} = e^{-\lambda\Omega \cdot t}$. The damping ratio ϵ possibly being very weak, nothing indicates a priori that we can neglect q_2 in relation to q_1. Unfortunately, keeping the exact values of θ_3 and ψ_3, the study of the stability must be much more complicated.

Reference

[1] Kane, T. R. and Barba, P. M., "Effects of energy dissipation on a spinning satellite," AIAA J. 4, 1391–1394 (1966).

Reply by Authors to Y. Pironneau

T. R. Kane* and P. M. Barba†
Stanford University, Stanford, Calif.

Pironneau's elegant solution of Eqs. (7) and (8) can be used to show that both $\theta_3' - \alpha$ and $\psi_3' - \alpha$ can be made arbitrarily small for all τ by a suitable choice of initial values of θ_3, ψ_3, θ_3', and ψ_3'. It is precisely this fact that justifies use of Eqs. (9) in conjunction with Eqs. (3–6) to study the stability problem posed in the paper, because only θ_3' and ψ_3' (but not θ_3 and ψ_3) appear in Eqs. (3–6). [This also applies to the full, nonlinear equations from which Eqs. (3–6) were derived.] In other words, our analysis does, in fact, take the complete solutions of Eqs. (7) and (8) into account.

Received March 6, 1967.
* Professor of Engineering Mechanics. Member AIAA.
† Graduate Student; presently Senior Engineering Specialist, Philco-Ford Corp. Member AIAA.

Comments on Precursors Ahead of Pressure Driven Shock Waves

Helmut D. Weymann*
University of Rochester, Rochester, New York

Nomenclature

h = Planck's constant
k = Boltzmann's constant
k_ν = absorption coefficient
n = electron or ion number density
t = time
u_s = shock velocity
x = distance ahead of shock front
z = distance between measuring station and diaphragm station
z_c = minimum of z for which a stationary precursor profile can be observed
N = photon flux density, $N_0 = N$ at $x = 0$
T = absolute temperature
α = absorption coefficient
ν = frequency

Received December 12, 1966.
* Professor of Mechanical and Aerospace Sciences.

Recently, W. G. Zinman[1] has discussed a possible explanation of the plasma density profiles ahead of pressure driven shock waves. Zinman based his theory on microwave measurements of Lederman and Wilson,[2] who found that the electron density profiles do not depend on the driven gas pressure and are stationary in a coordinate system moving with the shock front. These results are contrary to experimental findings of L. B. Holmes,[3] who used a collecting probe. In extensive investigations of the electron and ion densities in the precursor region, Holmes found that for Mach numbers ranging from 9 to 10.6, precursors propagating into argon at pressures between 2.5 and 10 torr do not become stationary in a shock fixed coordinate system up to diaphragm-probe distances of 310 cm. In addition, Holmes found that the plasma density at a given distance ahead of the shock front depends strongly on the driven gas pressure.

To explain the fact that their electron density profiles are independent of the gas pressure, Lederman and Wilson assumed that the atoms that are ionized in the precursor region are oxygen impurities whose partial pressure is independent of the gas pressure. Zinman, however, assumes that argon atoms are involved in at least the first step of excitation to a resonance level. He further assumes that the e-folding length of the precursor profile is inversely proportional to the distance ahead of the shock front. This is not born out by Lederman and Wilson's experiments.

Zinman's theoretical analysis is based on Wetzel's[4] treatment of photoionization in the precursor region. Unfortunately, Zinman makes several errors in going from his first to his second integral. These errors invalidate Zinman's formal derivation. There are, however, other assumptions that should be challenged. It would take too long to discuss all of Zinman's assumptions in detail. This however, is not necessary. His main point seems to be that he wants to show that $k_{\nu_{\text{eff}}} \approx 1/x$ and that this makes the electron density profiles independent of the pressure. In the last paragraph of the first column Zinman states that the k_ν range of interest (for the second integral) is $k_\nu \approx 1/x$. This expression is later pulled out of the calculation as proven. In addition to the fact that Zinman has not proven that the range around $k_\nu \approx 1/x$ gives the major contribution to the integral, even his a priori assumption that $k_\nu \approx 1/x$ cannot be made for arbitrary frequency dependences of the radiant flux and the absorption coefficient. For the case of one-step photoionization, one can show that the major contribution to Wetzel's Eq. (19) comes from the range around

$$k_\nu \approx (5/x^{3/4})(3k_\nu{}^0 kT/h\nu_i)^{1/4}$$

For $h\nu_i/kT = 15$ and $k_\nu{}^0 = 5\ 10^4\ \text{cm}^{-1}$, this expression becomes

$$k_\nu \approx 50/x^{3/4}$$

As k_ν is proportional to the density of absorbers, a change in the driven gas density will change the range of ν which, for a given x, will give the major contribution to Wetzel's Eq. (19). The magnitude of this contribution will change with the pressure because the photon flux density is a function of ν. For this reason there should be a pressure dependence of the plasma density profiles.

Finally, a comment will be made on the time necessary to establish a stationary plasma density profile relative to the shock front. As the driven gas is initially un-ionized, there must be a transient during which the stationary distribution is established. To find a lower limit for the distance which the shock has to travel until the plasma density at a distance x ahead of it has reached a constant value, let us assume that the photon flux density of the ionizing radiation is given by

$$N = N_0 \exp(-\alpha x)$$

with $x = z - u_s t$ and that the ionization rate is

$$dn/dt = \alpha N = \alpha N_0 \exp[-\alpha(z - u_s t)]$$

Integration of these two equations gives

$$n = (N_0/u_s)[\exp(-\alpha x) - \exp(-\alpha z)]$$

The plasma density at the distance x ahead of the shock front has reached a stationary state when the last term in the brackets can be neglected, compared with the first one. For the experimental conditions of Ref. 2 ($1/\alpha \approx 20$ cm) the plasma density has reached 90% of its final value when

$$z_c - x > 2.3/\alpha \approx 46 \text{ cm}$$

This means that the two probes will measure a stationary state at $x = 50$ cm ahead of the shock front if the distance of the first probe from the diaphragm station is larger than $z_c = 96$ cm.

In reality, z_c will be even larger because the shock itself must move a certain distance before it is fully developed, and before the region behind it starts to emit ionizing radiation. More complicated ionization mechanisms than the one assumed here should also increase z_c.

In Ref. 2 the diaphragm-probe distance is not given. However, the over-all length of the driven section is only 180 cm and one may estimate from the description of the shock tube that the first microwave cavity is not much more than 100 cm from the beginning of the driven section. It is, therefore, quite curious that Lederman and Wilson find stationary profiles in their shock tube.

References

[1] Zinman, W. G., "Comment on experimental precursor studies," AIAA J. **4,** 2073–2075 (1966).

[2] Lederman, S. and Wilson, D., "Microwave resonance cavity measurement of shock produced electron precursors," AIAA Paper 66-175 (March 1966); also AIAA J. **5,** 70–77 (1967).

[3] Holmes, L. B., "Plasma density ahead of pressure driven shock waves," Ph.D. thesis, Univ. of Rochester, 1965; also, Department of Mechanical and Aerospace Sciences, Univ. of Rochester, TN 1 (May 1965).

[4] Wetzel, L., "Far-flow approximations for precursor ionization profiles," AIAA J. **2,** 1208–1213 (1964).

Reply by Author to H. D. Weymann

W. G. Zinman*
Grumman Aircraft Engineering Corp., Bethpage, N. Y.

I WOULD like to thank Professor Weymann for his comments. I agree that the derivation of the second integral from the first integral should have been indicated in my comment[1]:

$$q(k_\nu) = q(\nu) = 1$$

and $$N(k\nu) = \int_0^{V_{\text{ionization}}} N(\nu)\delta[k_\nu(\nu)]d\nu$$

where δ is the Dirac delta function. Every resonance line for which $k_\nu^0 > k_\nu$ makes a contribution to the foregoing Stieltjes integral for $N(k\nu)$. Figures 17 and 20 of Lederman and Wilson's work[2] show that the l folding length is independent of pressure, a result not in accordance with Wetzel's model.[3] The assumption of steady state is reasonable since the diaphragm to probe distance is about 2 m[4] whereas the apparent photon near free path is about 20 cm.

I hope that Dr. Holmes will publish his results[5] and discuss the discrepancies between his results and those of Lederman and Wilson in more detail. There could be at least two significant factors. First, Lederman and Wilson conducted their work in a stainless-steel tube while that of Holmes was performed in a glass tube; second, there might be a difference in the impurity content of the argon used in the two investigations.

References

[1] Zinman, W. G., "Comment on experimental precursor studies," AIAA J. **4,** 2073–2075 (1966).

[2] Lederman, S. and Wilson, D., "Microwave resonance cavity measurement of shock produced electron precursors," AIAA Paper 66–175 (1966); also AIAA J. **5,** 70–77 (1967).

[3] Wetzel, L., "Far-flow approximations for precursor ionization profiles," AIAA J. **2,** 1208–1213 (1964).

[4] Wilson, D., unpublished communication (1966).

[5] Holmes, L. B., "Plasma density ahead of pressure driven shock waves," Ph.D. thesis, Univ. of Rochester (1965); also Department of Mechanical and Aerospace Sciences, Univ. of Rochester, TN1 (May 1965).

Received January 12, 1967.

* Propulsion Engineer, Propulsion Section; formerly Research Group Leader, Department of Aerospace Engineering and Applied Mechanics, Polytechnic Institute of Brooklyn, Farmingdale, N.Y. Member AIAA.